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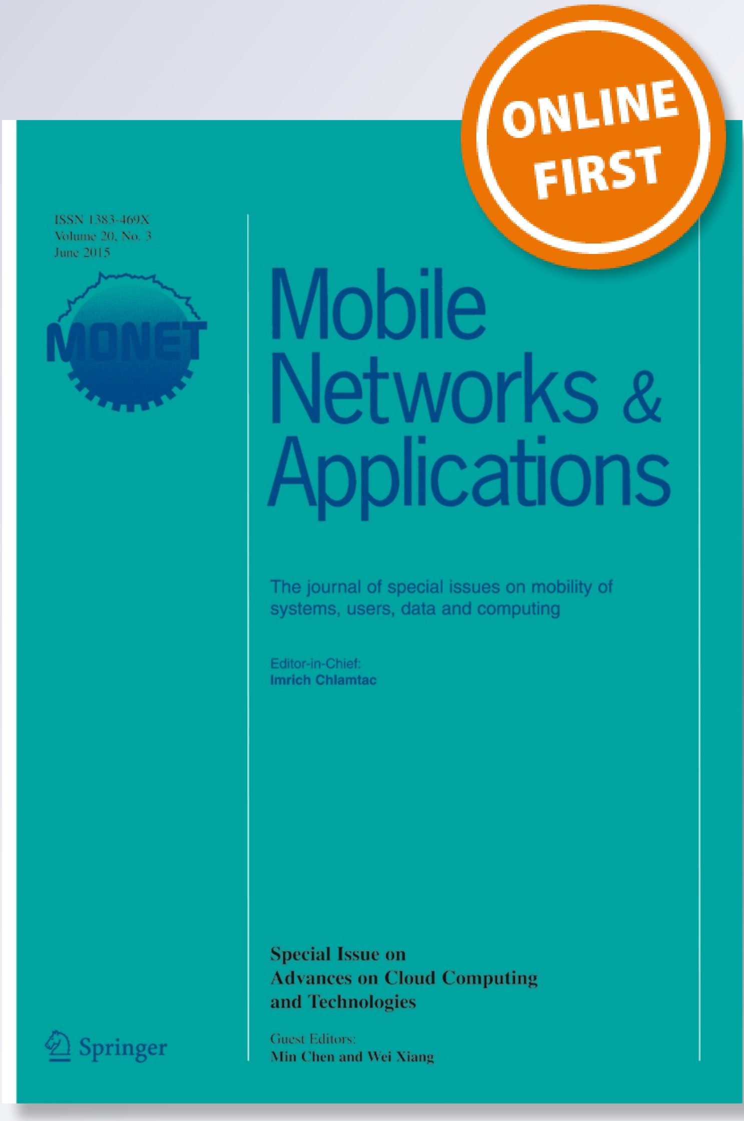

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A Pricing Power Control Scheme with Statistical Delay QoS Provisioning in Uplink of Two-tier OFDMA Femtocell Networks

Shenghua He[1] · Zhaoming Lu[1] · Xiangming Wen[1] · Zhicai Zhang[1] · Jun Zhao[1] · Wenpeng Jing[1]

Abstract Femtocell is a promising technique to enhance indoor coverage and improve network capacity. Nevertheless, because of the random and co-channel deployment of femtocells, the macrocell will suffer serious cross-tier interference from femtocells in two-tier femtocell networks. Thus, interference mitigation in femtocell networks has been an indispensable task. Meanwhile, with the explosive popularity of smart terminals, especially smart phones and tablets, the wireless networks have loaded a mount of data services with diverse delay quality of service (QoS) requirements. However, due to the stochastically varying nature of wireless physical channel, it is extremely difficult to offer a deterministic delay guarantee in wireless networks. Therefore, the effective capacity of femtocell users (FU) has been introduced to provide a statistical delay QoS provisioning. For that reason, in this paper, we will study the interference mitigation with statistical delay QoS guarantee in uplink two-tier orthogonal frequency division multiple access (OFDMA) femtocell networks. In order to mitigate the cross-tier interference at macrocell base station (MBS), we adopt a price-based power control strategy, in which the MBS protects itself by pricing the interference from FU. Additionally, to guarantee the statistical delay QoS for each FU, effective capacity is introduced into their utility functions. Then, a Stackelberg game is formulated to study the joint utility maximization of the MBS and the FUs subject to a maximum tolerable interference power constraint at the MBS. Subsequently, based on the mathematical analysis of the equilibrium of the formulated Stalkeberg game, a particle swarm optimization (PSO) aided power allocation (PSOPA) algorithm is proposed to solve this optimization problem. At last, simulation results show that our proposed PSOPA algorithm can not only improve significantly the average effective capacity of each FU and guarantee their statistical delay QoS, but also converge successfully.

Keywords Femtocell · Interference mitigation · Delay QoS · Effective capacity

1 Introduction

By the end of 2014, the number of mobile-connected devices will exceed the number of people on earth, and by 2018 there will be nearly 1.4 mobile devices per capita [1], which will cause explosive wireless data explosion in wireless networks. In addition, studies on wireless usage show that more than 50 percent of all voice calls and more than 70 percent of data traffic originate indoors [2]. To keep pace with this data explosion, femocell base station (FBS) has been put forward by the industry as a cost-effective means of offloading the macrocell network [3].

Femtocells, also called home base stations (HBS), are short-range, low-cost and low-power BSs which are installed by the consumer for better indoor voice and data reception, and they communicate with the cellular network over a broadband connection such as digital subscriber line (DSL), cable modem, or a separate radio frequency (RF) backhaul channel [2]. They provide operators with a promising technique to enhance the indoor coverage with very litter cost, which not merely improves network capacity but also prolongs the life of phone battery.

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Generally speaking, it is more practical for operators to deploy two-tier femtocell networks, which consists of femtocells deployed in macrocell networks, by sharing the spectrum, because of the scarce availability of spectrum resources and the absence of coordination on spectrum allocation between the macrocell and femtocells. However, due to the random and co-channel deployment of femtocells, the macrocell will suffer serious cross-interference in two-tier femtocell networks, which will greatly restrict the performance of the wireless network. Therefore, interference mitigation in femtocell networks has been an indispensable task since they were proposed. A great deal of scholarly works related to interference mitigation have recently appeared in the literature on design of spectrum-sharing femtocell networks. In [4], the author proposes a fully decentralized method for interference mitigation based on the observation of the signal to interference plus noise ratio (SINR) of all active communications in both macro and femtocells. In [5], inspired by the spirit of cognitive radio technology to mitigate detractive interference, a cognitve radio resource management (CRRM) scheme has been proposed. Paper [6] treats the uplink interference problem in OFDMA-based femtocell networks with partial cochannel deployment. In [7], the author derives the per-tier outage probability by introducing a simplified mathematics model that provides closely approximate femtocell interference distribution. In [8], the author considers a distributed power control strategy, modeled as a noncooperative game, where users maximize their utilities in a multicell system. Admittedly, the strategies proposed in [4–8] can mitigate cross-tier interference effectively, however, both of them haven't considered the QoS guarantee in femtocell networks.

Related works in consideration of QoS guarantee for femtocell users have been studied in [9–14]. In [9], the author analyzes the power consumption and QoS support levels, which is represented by the average transmission bandwidth, in a femtocell network. In [10], a spectrum splitting methods with QoS-oriented fairness metric provisioning, which considers the ratio of the sum capacity of the macrocell to the sum in femtocell network has been proposed. In [11], a new QoS management scheme, which translates the QoS requirements into appropriate number of slots within the WiMax frame, has been proposed for WiMAX Femtocell Access Point (WFAP). In paper [12], the author studies the admission control for femtocell communications by predicating the QoS metrics such as network loads/congestion indications and QoE metrics. In [13], the outage probability has been taken into consideration as the QoS metric in the interference management scheme. In paper [14], the power control strategy via game theoretic approach has been studied, which considers the macrocell users' QoS in terms of SINR. In conclusion, the QoS metrics involved in lecture [9–14] generally include the SINR, data rate, outage probability, or the load/congestion indications, and so on, but the delay QoS has been seldom considered.

Actually, with the popularity of the smart mobile terminals, such as smart phones and tablets, the wireless networks have loaded a mount of data services with diverse delay QoS requirements. For instance, a mixture of delay sensitive applications (e.g. video teleconferencing) and delay tolerant ones (e.g., web browsing and file downloading) must be supported in order to provide the users with good experience. Therefore, delay QoS guarantee for high-data-rate services in two-tier femtocell networks has recently become a increasingly significant and challenging research area.

However, due to the time-varying nature of wireless channels, it is quite difficult to impose a deterministic delay guarantee for services over wireless networks. To address this issue, the concept of effective capacity has been proposed [15], which is defined as the maximum constant arrival rate that a wireless physical channel can support to guarantee the statistical delay requirements. In paper [16–18], the effective capacity has been widely adopted to provide the statistical delay provisioning to ensure a small steady-state delay violation probability. But these power allocation strategies provisioning statistical delay QoS in [16–18] are mainly aiming at a single cell, which is extremely different from the practical two-tier femtocell networks.

In this paper, we consider the uplink of the OFDMA two-tier femtocell networks, with femtocells overlaying the macrocell. Here, we talk about the system capacity provisioning statistical delay QoS, which can be modeled as effective capacity. If a FU individual transmits data at a higher power, its effective capacity will be higher, but the interference to the macrocell will get larger. As there is a threshold at the macrocell base station, the transmit power should not be overlarge in order to guarantee the communication quality in the macrocell networks. Therefore, in order to mitigate the cross-tier interference from FU to the MBS and provide a statistical delay QoS guarantee for all FUs, we propose a power allocation algorithm named PSOPA, which can enhance significantly the system's effective capacity.

The contributions of this paper are summarized as follows:

(1) In order to mitigate the cross-tier interference at the MBS side, this paper proposes a pricing power control strategy, in which the MBS protects itself by pricing the interference from femtocell users. In addition, the MBS restricts its received interference power to a maximum tolerable margin at the MBS receiver.

(2) In this paper, we adopt the effective capacity in the utility functions of femtocell users to ensure a small steady-state delay violation probability and good delay-QoS guarantee.

(3) We formulate the power allocation problem as a Statelberg game to jointly maximizes the revenue at the MBS and the effective capacity of the femtocell users, under the given QoS requirements. In this game, the MBS is the leader and the femtocell users are the followers. The leader (MBS) sets prices firstly, and the followers (FUs) adjust their power strategies based on the given prices. The Statelberg equilibrium has been analyzed mathematically in sparsely and densely deployed scenarios, respectively. Subsequently, PSO algorithm has been used to search the equilibrium point and an efficient power control algorithm, named PSOPA, has been proposed. The basic idea for this power control algorithm can be briefly concluded into the following two layers:

- **outer-layer:** PSO-based search for the optimal interference price;
- **inner-layer:** for a given price, each FU chooses a proper transmit power.

The remainder of this paper is organized as follows. In Section 2, we describe the system model and formulate the problem as a pricing Stackelberg game. Subsequently, in section 3, the Statelberg equilibrium has been analyzed mathematically in sparsely deployed and densely deployed scenarios, respectively, and a PSOPA algorithm has been proposed to obtain the SE point. In Section 4, simulation results are presented to analyze the performance of the proposed algorithm. Finally, we conclude the paper in Section 5.

2 System model and game formulation

In this section, firstly, we describe the system model of the two-tier femtocell network and introduce the concept of effective capacity. Secondly, we formulate the power allocation problem as a Stalkberg game and investigate its equilibrium.

2.1 Network description

We consider the uplink of a two-tier OFDMA femtocell network, where K femtocells are underlaying in a macrocell. To take advantage of the spectrum, all the femtocells and the macrocell will share the same spectrum. Obviously, there will exist co-channel interference, such as cross-tier interference from the FUs to the macrocell and intra-tier interference between each femtocell in the network.

We assume that a femtocell will independently allocate its subchannels, and there is only one scheduled active user during each time slot. As the subchannels are orthogonal between each other in OFDMA networks, we assume that each subchannel can be allocated independently among the scheduled active FUs at each time slot. Hence, in this paper, we study the power allocation on a single subchannel in uplink of the femtocell network, but it is worth pointing out that this assumption can be easily extended to broadband femtocell systems with parallel frequency subchannels. As a result, this two-tier femtocell network can be simplified as Fig. 1 shows. In addition, for the purpose of exposition, all channels involved are assumed to be independently block-fading, which means that the channel gains retain constant during each transmission block, but vary from one block to another.

Let B_k denote FBS k, where $k \in \mathcal{K} = \{1, 2, ..., K\}$. FU k represents the scheduled active user serviced by B_k and its transmit power is p_k. Let $h_{k,j}$ and g_k be instantaneous channel power gains from FU k to B_j and from FU k to the MBS respectively. We assume that, the additional interference at the B_k's side from the macrocell users is regarded as the background noise, which is independent Gaussian random process with zero mean and variance σ^2.

The SINR of FU k on the considered subchannel is expressed as

$$\gamma_k(p_k, \boldsymbol{p_{-k}}) = \frac{p_k h_{k,k}}{\sum\limits_{j \neq k} p_j h_{j,k} + \sigma^2}, \forall k \in \mathcal{K}, \tag{1}$$

where $\boldsymbol{p_{-k}}$ denotes other FUs' transmit power except FU k.

According to the Shannon's capacity formula, the ideal achievable data rate of FU k is

$$R_k(p_k, \boldsymbol{p_{-k}}) = w log_2(1 + \gamma_k(p_k, \boldsymbol{p_{-k}})), \tag{2}$$

where w is the bandwidth of the subchannel.

To guarantee the reliability of the macrocell, at the MBS side, the received cross-tier interference should be restricted to a certain threshold. We assume that the maximum interference the MBS can tolerate is I_{max} dBm, then the following constraint should be satisfied at MBS side:

$$\sum_{k=1}^{K} g_k p_k \leq I_{\max}, \tag{3}$$

2.2 Effective capacity (EC)

As the explosion of the time-sensitive services in the wireless networks, it is more and more vital to take the delay-QoS metric into consideration when allocating the wireless resources. For the purpose of improving the experience for each user, in this paper, each femtocell should provide heterogeneous delay-QoS services for its FUs. However, as the result of the time varying nature of physical wireless channel, $R_k(p_k, \boldsymbol{p_{-k}})$ is stochastically varying, which makes it extremely difficult to provide an accurate delay bound guarantee for FU k. Therefore, we introduce the statistical QoS metric and delay-bound violation probability to describe the delay-QoS for each FUs.

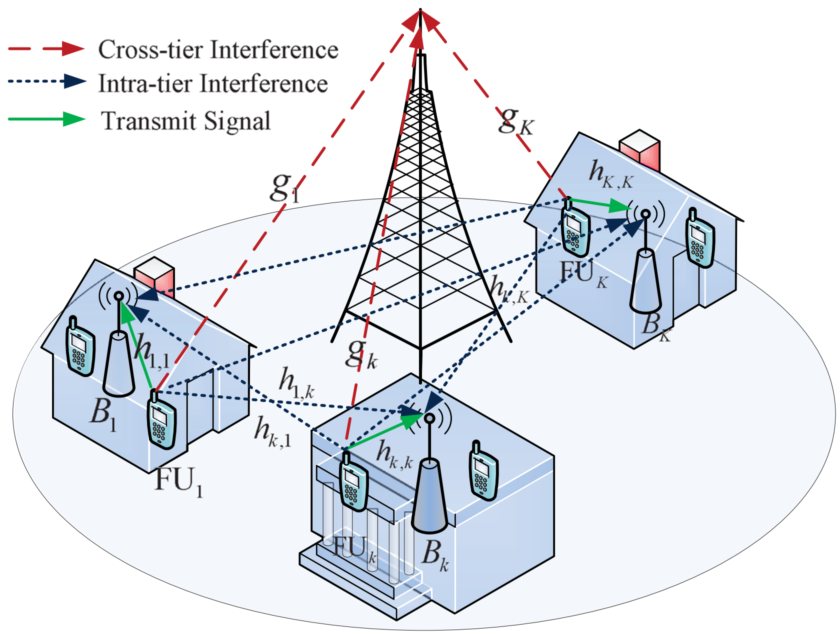

Fig. 1 The system model of the two-tier femtocell network

We assume that the data will be put into a first-in-first-out (FIFO) buffer before being transmitted in the physical channel. According to [10], the steady-state delay violation probability of FU k is

$$P\{D_k \geq D_k^{\max}\} \approx e^{-\theta_k c_k D_k^{\max}}, \tag{4}$$

where D_k represents the actual delay and is a random variable, $D_k^{\max}$ is the delay bound, θ_k is the statistical delay exponent of the FU k and c_k is a constant determined by the arrival and service processes. For a given $D_k^{\max}$, a smaller θ_k leads to a larger $P\{D_k \geq D_k^{\max}\}$, which implies a looser delay-QoS constraint. Otherwise, a larger θ_k corresponds to a more stringent delay-QoS constraint.

After taking the statistical QoS metric into consideration, EC has been proposed to describe the arrival data which considers the delay-QoS requirements. It is defined as the maximum constant arrival data rate at the link layer that a given service process (wireless physical channel) can support under the given statistical delay requirement specified by θ_k [18]. Analytically, the EC of FU k is given by

$$E_c(\theta_k) = -\frac{1}{\theta_k T} \ln \left\{ \mathbb{E}\left[e^{-\theta_k T R_k(p_k, p_{-k})} \right] \right\}, \tag{5}$$

where $\mathbb{E}$ is the expectation operator and T is the block duration. It is obvious that as θ_k gets larger, which means a stringent delay-QoS constraint, the EC of FU k decreases. The effective capacity model can be simply described in Fig. 2.

2.3 Stackelberg game formulation

Stackelberg game is a strategic game that consists of a leader and several followers competing with each other on certain resources. The leaders have the priority to make their decisions, and the followers do subsequently [19, 20]. Both of them take action rationally and selfishly to improve their own revenues.

In this paper, we formulate the power allocation problem as a Stackelberg game, in which the MBS is the leader and protects itself by imposing a set of prices on per unit of received interference power from each FU, and the FUs are the followers, which update their power allocation strategies to maximize their individual EC based on the assigned interference prices.

2.3.1 The utility function at the FU side

At FU's side, definitely, each FU expects to have a much higher effective capacity. Considering that $\ln(x)$ is a monotonically increasing function, from Eq. 5, a higher $E_c(\theta_k)$ means a larger function value of $-\mathbb{E}\left[e^{-\theta_k T R_k(p_k, p_{-k})}\right]$. Therefore, the utility for FU k can be formulated as

$$U_k(p_k, p_{-k}) = -\mathbb{E}\left[e^{-\theta_k T R_k(p_k, p_{-k})} \right] - \lambda_k I_k(p_k), \tag{6}$$

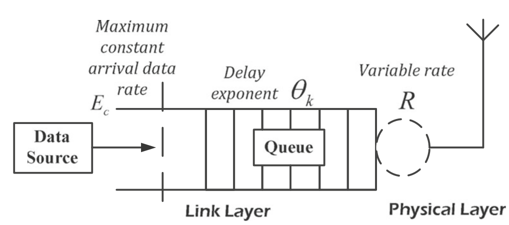

Fig. 2 The Effective Capacity Model

where $I_k(p_k) = g_k p_k$ is the interference quota FU k would like to buy from the MBS under the interference price λ_k. It is observed from Eq. 6 that the utility function of each FU consists of two parts: the profit, represented by the first part, and the cost, described by the second part. If the FU k increases its transmit power, the profit increases, which implies larger effective capacity, but it will definitely cause more interference to the MBS and needs to pay for it at the cost of $\lambda_k I_k(p_k)$. For each FU k, this problem can be formulated as

$$\textbf{Problem2.1}: \quad \max U_k(p_k, p_{-k}), \tag{7a}$$
$$p_k \geq 0, \forall k \in \mathcal{K} \tag{7b}$$
$$p_k \leq p_{\max}, \forall k \in \mathcal{K} \tag{7c}$$
$$\theta_k > 0, \forall k \in \mathcal{K}, \tag{7d}$$

where p_{max} is the maximum transmit power of FU k on this subchannel.

2.3.2 The utility function at the MBS side

At the MBS's side, if the price it imposes on each FU is too high, it would happen that no FU could afford the interference quota and choose to transmit data on the given subchannel, which would lead to the decrease of the spectrum efficiency. In another extreme, if the price is too low, each FU would be happy to increase their power as high as possible, which would also cause severe cross-tier interference to the MBS in the uplink. Thereby, at the MBS's side, its intention is to maximize its revenue expressed by

$$U_M(\lambda, p) = \sum_{k=1}^{K} \lambda_k I_k(p_k), \tag{8}$$

where $\lambda = [\lambda_1, \lambda_2, ..., \lambda_K]^T$ represents the interference price vector, and λ_k denotes the price that is set on the interference from FU k. $\boldsymbol{p} = [p_1, p_2, ..., p_K]^T$ is power allocation matrix for all the FUs. In addition, Eq. 3 should also be satisfied in order to restrict total interference at the MBS. Therefore, the problem should be searching the optimal interference prices $\boldsymbol{\lambda}$ to maximize its revenue within its tolerable aggregate interference margin, I_{max}. The problem can be formulated as:

$$\textbf{Problem2.2}: \quad \max U_M(\lambda, \boldsymbol{p}), \tag{9a}$$
$$\sum_{k=1}^{K} I_k(p_k) \leq I_{max}, \tag{9b}$$
$$\lambda > 0. \tag{9c}$$

Based on the analysis above, Problem 2.1 and Problem 2.2 together form a Stackelberg game. In this game, all the players take action selfishly and rationally. The leader (MBS) anticipates to adjust the price $\boldsymbol{\lambda}$ to improve its utility, and each follower (FU) is eager to increase its transmit power to boost its EC. The leader take action firstly, and the followers take action subsequently. The ultimate objective of this game is to attain the SE point(s), from which neither the MBS nor each of the FUs has the incentive to deviate.

2.4 Stackelberg equilibrium

For the proposed Stackelberg game, the SE is defined as follows.

Definition 2.1 Let $\boldsymbol{p}^* = (p_1^*, p_2^*, ..., p_K^*)$ be a solution for Problem 2.1 at all the FUs and $\lambda^* = (\lambda_1^*, \lambda_2^*, ..., \lambda_K^*)$ be a solution for Problem 2.2. Then the point $(\lambda^*, \boldsymbol{p}^*)$ is a SE for the proposed Stackelberg game if for any $(\lambda, \boldsymbol{p})$, the following conditions are satisfied:

$$U_M(\lambda^*, \boldsymbol{p}^*) \geq U_M(\lambda, \boldsymbol{p}^*), \tag{10}$$

$$U_k(p_k^*, \boldsymbol{p}_{-k}^*, \lambda^*) \geq U_k(p_k, \boldsymbol{p}_{-k}^*, \lambda^*). \tag{11}$$

It is not difficult to see that for a given $\boldsymbol{\lambda}$, all the FUs strictly compete in a non-cooperative manner, which can be seen as the non-cooperative subgame. Therefore, each round of the Stackelberg game can be conducted as the following two main steps: firstly, the leader will take action and set a price set for the followers; secondly, the followers observe the leader's strategy and compete in a non-cooperative manner, which can be described as

$$\mathcal{G} = \{\mathcal{K}, \{\mathcal{P}_k\}_{k\in\mathcal{K}}, \{U_k\}_{k\in\mathcal{K}}\}, \tag{12}$$

where $\mathcal{K}$ denotes the set of all the active FUs, $\mathcal{P}_k$ denotes FU k's power strategy space, and U_k is FU k's utility function, which is expressed as Eq. 6.

Generally, the SE for a Stackelberg game can be obtained by finding its subgame's Nash Equilibrium (NE) point firstly. Then, the best response of the MBS will be readily obtained by solving Problem 2.2 based on the subgame's NE solution. Thus, the SE can be obtained as follows. For a given $\boldsymbol{\lambda}$, Problem 2.1 is solved firstly. Then, based on the obtained best response functions $\boldsymbol{p}^*$ of the FUs, which is a function of $\boldsymbol{\lambda}$, we solve Problem 2.2 for the optimal interference price $\boldsymbol{\lambda}^*$.

3 Power allocation algorithm

In this section, we provide the solutions for the formulated problems with the following steps: analyze the utility of each FU mathematically and find the NE for the non-cooperative subgame firstly, and search the SE point subsequently. For expositive simplicity, we assume that the MBS gives interference price for each FU uniformly, namely $\lambda_1 = \lambda_2 = \ldots = \lambda_K = \lambda$, where λ is the uniform interference price. Actually, the solution also suit for the non-uniform price case.

3.1 NE of the non-cooperative subgame

3.1.1 Sparsely deployed scenario

Firstly, we consider the sparsely deployed scenario, i.e., in rural areas, where the interference between each femtocell can be negligible, i.e., $h_{j,k} = 0, \forall j \neq k$. As a result, the Problem 2.1 can be translated into

$$\textbf{Problem3.1}: \quad \max \ \mathbb{E}\left[-e^{-\theta_k wT \log\left(1+\frac{h_{k,k}p_k}{\sigma^2}\right)}\right] - \lambda g_k p_k, \tag{13a}$$

$$p_k \geq 0, \forall k \in \mathcal{K} \tag{13b}$$

$$p_k \leq p_{\max}, \forall k \in \mathcal{K} \tag{13c}$$

$$\theta_k > 0, \forall k \in \mathcal{K}. \tag{13d}$$

Theorem 1 *For a given interference price λ, $U_k(p_k, \boldsymbol{p_{-k}})$ is a concave function over p_k, which has a unique optimal solution as*

$$\widehat{p}_k(\lambda) = \frac{\sigma^2}{h_{k,k}}\left[\left(\frac{\theta_k \beta h_{k,k}}{\lambda g_k \sigma^2}\right)^{\frac{1}{1+\theta_k\beta}} - 1\right]. \tag{14}$$

where $\beta = \frac{wT}{\ln 2}$.

Proof In this paper, we assume that the $f(h_{k,k})$ represents the probability density function (PDF) of $h_{k,k}$. Then $U_k(p_k, \boldsymbol{p_{-k}})$ can be represented as

$$\begin{aligned} U_k(p_k, \boldsymbol{p_{-k}}) &= \mathbb{E}\left[-e^{-\theta_k wT \log_2\left(1+\frac{h_{k,k}p_k}{\sigma^2}\right)}\right] - \lambda g_k p_k \\ &= \int\left[-e^{-\theta_k wT \log_2\left(1+\frac{h_{k,k}p_k}{\sigma^2}\right)} - \lambda g_k p_k\right] f(h_{k,k}) dh_{k,k} \end{aligned} \tag{15}$$

The first and second derivates of $U_k(p_k, \boldsymbol{p_{-k}})$ are respectively

$$\begin{aligned} \frac{\partial U_k(p_k, \boldsymbol{p_{-k}})}{\partial p_k} &= \mathbb{E}\left\{\frac{\partial}{\partial p_k}\left(-e^{-\theta_k wT \log_2\left(1+\frac{h_{k,k}p_k}{\sigma^2}\right)} - \lambda g_k p_k\right)\right\} \\ &= \mathbb{E}\left\{\frac{\partial}{\partial p_k}[-\left(1+\frac{h_{k,k}p_k}{\sigma^2}\right)^{-\theta_k\frac{wT}{\ln 2}} - \lambda g_k p_k]\right\} \\ &= \mathbb{E}\left\{\frac{h_{k,k}\theta_k}{\sigma^2}\frac{wT}{\ln 2}\left(1+\frac{h_{k,k}p_k}{\sigma^2}\right)^{-\left(\theta_k\frac{wT}{\ln 2}+1\right)} - \lambda g_k\right\} \\ &= \int [\frac{h_{k,k}\theta_k}{\sigma^2}\beta\left(1+\frac{h_{k,k}p_k}{\sigma^2}\right)^{-(\theta_k\beta+1)} - \lambda g_k] f(h_{k,k}) dh_{k,k} \end{aligned} \tag{16}$$

and

$$\begin{aligned} \frac{\partial^2 U_k(p_k, \boldsymbol{p_{-k}})}{\partial^2 p_k} &= \frac{\partial}{\partial p_k}\left[\frac{\partial U_k(p_k, p_{-k})}{\partial p_k}\right] \\ &= -\int\left[\theta_k(\theta_k\beta+1)\frac{h_{k,k}^2}{\sigma^4}\beta(1+\frac{h_{k,k}p_k}{\sigma^2})^{-(\theta_k\beta+2)}\right] f(h_{k,k}) dh_{k,k}, \end{aligned} \tag{17}$$

where $\beta = \frac{wT}{\ln 2}$.

As Eq. 17 shown, $\frac{\partial^2 U_k(p_k, \boldsymbol{p_{-k}})}{\partial^2 p_k} \leq 0$ is satisfied at any p_k in its solution space. Thus, the objective function is a concave function over p_k and has a unique optimal solution. □

According to the theory in [21], for a given interference price λ, the optimal solution for Eq. 15 should satisfy $\frac{\partial U_k(p_k, \boldsymbol{p_{-k}})}{\partial p_k} = 0$. Then merely optimal solution is given by

$$\widehat{p}_k(\lambda) = \frac{\sigma^2}{h_{k,k}}\left[\left(\frac{\theta_k \beta h_{k,k}}{\lambda g_k \sigma^2}\right)^{\frac{1}{1+\theta_k\beta}} - 1\right]. \tag{18}$$

Thus, Theorem 1 is proved.

Theorem 1 demonstrates the existence and uniqueness of the global optimal solution in $U_k(p_k, \boldsymbol{p_{-k}})$. Therefore, solution of Problem 3.1 is accordingly

$$p_k^*(\lambda) = \begin{cases} \min\{\widehat{p}_k(\lambda), p_{max}\}, & \lambda < \frac{\theta_k\beta h_{k,k}}{g_k\sigma^2} \\ 0 & , \lambda \geq \frac{\theta_k\beta h_{k,k}}{g_k\sigma^2} \end{cases}. \tag{19}$$

It is observed that for a given interference λ, the NE for the subgame at FU side can be easily obtained at $p_k^*(\lambda)$. If the interference price is too high, i.e., $\lambda \geq \frac{\theta_k\beta h_{k,k}}{g_k\sigma^2}$, FU k will not transmit data on this subchannel. In other extreme, FU k will translate at p_{max}.

3.1.2 Densely deployed scenario

In this part, we consider the densely deployed scenario, i.e., in the urban area, where mutual interference between each femtocell can not be neglected. Based on the conclusion similar to Theorem 1, for given λ and $\boldsymbol{p}_{-k}$, the best response function of FU k can be obtained as

$$\widehat{p}_k(\lambda, \boldsymbol{p}_{-k}) = \frac{z^2(\boldsymbol{p}_{-k})}{h_{k,k}}\left[\left(\frac{\theta_k \beta h_{k,k}}{\lambda g_k z^2(\boldsymbol{p}_{-k})}\right)^{\frac{1}{1+\theta_k\beta}} - 1\right], \tag{20}$$

$$p_k^*(\lambda, \boldsymbol{p}_{-k}) = \begin{cases} \min(\widehat{p}_k(\lambda, \boldsymbol{p}_{-k}), p_{max}), & \lambda < \frac{\theta_k\beta h_{k,k}}{g_k z^2(\boldsymbol{p}_{-k})} \\ 0 & , \lambda \geq \frac{\theta_k\beta h_{k,k}}{g_k z^2(\boldsymbol{p_{-k}})} \end{cases}, \tag{21}$$

where $z^2(\boldsymbol{p}_{-k}) = \sigma^2 + \sum_{j \neq k} p_j h_{j,k}$.

In non-cooperative game, the formula (21) actually represents the strategy FU k would like to give based on all the other players' strategies, including λ and $\boldsymbol{p_{-k}}$.

Theorem 2 *A Nash equilibrium exists in game in* $\mathcal{G} = \{\mathcal{K}, \{\mathcal{P}_k\}_{k\in\mathcal{K}}, \{U_k\}_{k\in\mathcal{K}}\}$ *if the following conditions are satisfied.* $\forall k \in \mathcal{K}$,

(1) $\mathcal{P}_k$ *is a nonempty, convex and compact subset of some Euclidean space* $\Re^K$.

(2) $U_k(p_k, \boldsymbol{p}_{-k})$ *is continuous in* $\boldsymbol{p}$ *and quasi-concave in* p_k.

Obviously, the condition (1) in Theorem 2 is satisfied in game $\mathcal{G}$. Moreover, as is proved in Theorem 1, the $U_k(p_k, \boldsymbol{p}_{-k})$ is concave in p_k. Thus, Theorem 2 establishes the existence of NE point(s) in this non-cooperative power control subgame. Nevertheless, the monotonicity of $U_k(p_k, \boldsymbol{p}_{-k})$ in p_k is not certain in Eq. 6. According to the theorem in [22], there generally exist multiple NEs, but there is no efficient algorithm to obtain all of them.

Fortunately, since the inter-femtocell channel power gains are usually very weak due to the penetration loss, we can assume that the aggregate interference at B_k's receiver received from other femtocell co-channel FUs in uplink is bounded, i.e., $\sum\limits_{j\neq k} p_j h_{j,k} \leq \delta$, where δ is the upper bound. Here, we can merely consider the worst case, i.e., $\sum\limits_{j\neq k} p_j h_{j,k} = \delta, \forall k \in \mathcal{K}$. If we denote $\sigma^2+\delta$ as z^2, the problem at the FU side will be exactly the same as Problem 3.1 with σ^2 replaced by z^2. So the NE point is unique at this case and the NE can be calculated by formula (21).

3.2 The equilibrium of the Staclkberg game

As analyzed above, for a given λ, the NE of the non-cooperative subgame will be obtained under the sparsely deployed scenario and the densely deployed one with inter-femtocell interference seriously restricted. Substituting $p_k^*(\lambda)(k \in \mathcal{K})$ into Problem 2.2, the optimization problem at the MBS side can be formulated as

$$\textbf{problem3.2}: \quad \max \sum_{k=1}^{K} \lambda g_k p_k^*(\lambda) \tag{22a}$$

$$\sum_{k=1}^{K} g_k p_k^*(\lambda) \leq I_{\max}, \tag{22b}$$

$$\lambda > 0. \tag{22c}$$

By solving Problem 3.2, we will obtain the proper price, λ^*, which brings the largest revenue at the MBS side. After that, SE will be obtained when we substitute λ^* into Eq. 19 or Eq. 21.

In general, problem 3.2 is difficult to solve directly. In this paper, we choose to utilize the PSO algorithm to search the optimal price.

3.3 PSO aided power allocation algorithm

3.3.1 PSO (Particle Swarm Optimization)

PSO, firstly proposed by Dr. Kennedy in [23], is an evolutionary computation technique based on swarm intelligence. It utilizes a simple intelligent mechanism that mimics swarm behavior in birds flocking and fish schooling to guide the particles to search for globally optimal solutions. Owing to the bio-mimic hunting behaviors, PSO algorithm can search the optimal solution quite fast. In addition, the parameters in PSO algorithm are very little, which makes it very easy to operate. In consideration of these advantages, we will introduce it into our paper to assist searching the optimal solution for our proposed problems later.

Each individual in the swarm is a volume-less particle in solution search space, which contains two parameters: the position and the flying velocity. The position represents potential solution of optimization problem in search space, and the flying velocity determines the direction and step of the search at present. The particle flies in search space at a definite velocity which is dynamically adjusted by tracing the best position found so far by its own and that of the whole swarm at present. Particle swarm tracks the two best current positions, moves to better region gradually, and finally arrives at the best position of the whole search space [24, 25].

Denote the position and flying velocity of a particle as x and v, respectively. The performance of the position is evaluated by the fitness function, denoted as $\mathcal{F}$, using x as input. The higher the function value, the better the position. At each iteration, each particle records its personal optima, x_p, which represents the best position it has achieved so far. At the same time, the swarm records the best position, called as global optima and denoted as x_g. As shown in Fig. 3,

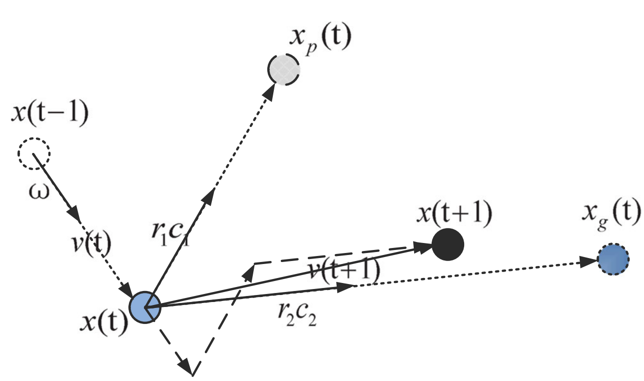

Fig. 3 The Update of Particle and Velocity

the velocity and position of a particle are updated at each iteration according to the formulas given below

$$v(t+1) = \omega v(t) + c_1 r_1 (x_p(t) - x(t)) + c_2 r_2 (x_g(t) - x(t)), \quad (23)$$

$$x(t+1) = x(t) + v(t+1), \quad (24)$$

where t represents iteration number; ω is inertia weight; r_1 and r_2 are random numbers independently and uniformly distributed in the range [0,1]; c_1 and c_2 are the learning factors, which are usually both set to 2.0 based on the trial results.

The first part of Eq. 23 is previous velocity of particle, reflecting the the particle's memory. The second part is cognitive action of particle, reflecting particle's thinking. The third part is social action of particle, reflecting information sharing and mutual cooperation between particles.

In addition, ω is the inertia weight to obtain tradeoff between the local and global research. In practice, at the beginning, to search the solution space expansively, the individual experience is important and the ω should be large. But when approaching to global optima, in order to accelerate the convergence, the inertia weight should be reducing to small degree. Thus, in this paper, we introduce a linear decreasing inertia weight which is described as

$$\omega(t) = \frac{t_{\max} - t}{t_{\max}} (\omega_{\max} - \omega_{\min}) + \omega_{\min}, \quad (25)$$

where, $\omega_{\max}$ and $\omega_{\min}$ are respectively the maximum and the minimum inertia weight; $t_{\max}$ is the maximum times of iteration.

After each update, they record the best position, and finally the particles swarm will find an optimal position in the solution space. The cessation condition of iteration is that the greatest iteration number arrives or the best previous position fits for the minimum adaptive value.

Usually, the convergence property of the PSO algorithm is correlating to population of the swarm, which is defined as the number of the particles in swarm. More the number, the more positions will be searched at one time. Then, at the same iteration number, the swarm with larger population will search more positions than the swarm with smaller one. Thus, in practice, a larger population in a swarm would lead to an more accurate search in PSO algorithm.

3.3.2 PSO aided power allocation

Based on the analysis above, we propose a PSO aided power allocation (PSOPA) algorithm, which would not only mitigate the cross-tier interference at the MBS from FUs, but also improve each FUs' EC. The basic idea can be concluded into the following two layers:

1) outer-layer: search for the optimal price λ with PSO; x is taken as λ^* and the fitness function is defined as $\mathcal{F}(\lambda^*) = \sum \lambda^* I_k(\lambda^*)$.

2) inner-layer: for a given price λ^*, each FU choose a transmit power with the conclusion (19) or (21).

The complete algorithm is shown in **Algorithm 1**.

Algorithm 1 PSO aided Power Allocation

Step 1: Input: $\theta = [\theta_k]_{1\times K}$, $\mathbf{h} = [h_k]_{1\times K}$, $\mathbf{g} = [g_k]_{1\times K}$, swarm size as M and calculate the largest $\lambda_{\max}$ with (19) to make at least one FU access.

Step 2: Particle swarm initiation

Denote the particle swarm as $\mathcal{M} = \{1, 2, \ldots, \mathrm{M}\}$. Their positions and velocities are respectively initialized as $\mathcal{R}(0) = \{\lambda_1(0), \lambda_2(0), \ldots, \lambda_M(0)\}$ and $\mathcal{V}(0) = \{v_1(0), v_2(0), \ldots, v_M(0)\}$, where $\forall m \in \mathcal{M}$, $\lambda_m(0)$ randomly distributed in $(0, \lambda_{\max})$ and satisfies (22b), and $v_m(0)$ is randomly and uniformly distributed in $(0, v_{\max})$.

Initialize each personal optima: $\lambda_{m,p}(0) = \lambda_m(0)$.

Initialize global optima: $\lambda_g(0) = arg \max_{\lambda \in \mathcal{R}(0)} F(\lambda)$

Step 3: Begin to search the global optima.

while$(t \leq t_{\max})$ **do**

Update the swarm's positions and velocities to $\mathcal{R}(t)$ and $\mathcal{V}(t)$ with Eqs. 23 and 24.

  for$(m = 1 : M)$

    if$(F(\lambda_m(t)) > F(\lambda_{m,p}(t-1)) \&\& I(\lambda_m(t)) < I_{max})$

    then$\lambda_{m,p}(t) = \lambda_m(t)$ **else** $\lambda_{m,p}(t) = \lambda_{m,p}(t-1)$

  end for

  $\lambda^* = arg \max_{\lambda \in \mathcal{R}(t)} F(\lambda)$

  if$(F(\lambda^*) > F(\lambda_g(t-1)))$

  then $\lambda_g(t) = \lambda^*$ **else** $\lambda_g(t) = \lambda_g(t-1)$

end while

Output the best position λ_g as the optimal price.

Step 4: Each FU's transmit power p_k^* will be calculated with (14) **or** (19)

4 Simulation and numerical analysis

In this section, we conduct several simulations to verify the performance of our proposed power allocation algorithm in two-tier OFDMA femtocell networks. Firstly, the convergence property of our proposed PSOPA algorithm has been shown. Secondly, the performance of the femtocell networks based on PSOPA is considered.

We simulate our algorithm on a MATLAB-programmed platform. In this platform, FBSes are randomly distributed within the coverage of a central MBS and the FUs are randomly located within the coverage of their belonged FBSes. The mainly scalable parameters are listed in Table 1 [26], where d represents the transmitter-receiver separation in meters and L_w is the penetration loss through external

Table 1 Simulation Parameters

Parameter	Value
Macrocell radius	500m
Femtocell radius	10m
Carrier frequency	2.5GHz
Subchannel Bandwidth	200KHz
Number of FBS	10-15
FU maximum power	23dBm
Background noise	−70dBm
Indoor path loss	$(38.46 + 20\lg d)$ dB
In-outdoor path loss	$(38.46 + 20\lg d + L_w)$ dB

walls assumed to 15dB here. Here, we mainly consider the sparsely deployed scenario, but the densely deployed one with inter-femtocell interference seriously restricted will be also simulated later.

4.1 Convergence property of PSOPA

In this subsection, we study the convergence property of our proposed PSOPA algorithm. Without loss of generality, the maximum interference the MBS can tolerate can be set to -85dBm. The number of the femtocells is 10. The delay-QoS exponent constraints for the FUs, which represent different delay requirements, are respectively set as $\theta_k = 10^{-3}(k = 1, 2, 3, 4, 5)$ and $\theta_m = 10^{-2}(m = 6, 7, 8, 9, 10)$.

During the simulation proceeding, we have set the population of swarm at different number. Take the swarm numbers as 6, 12 and 18 for representation and the respective results are shown in Fig. 4. It illustrates that our proposed PSOPA algorithm converges respectively at the 7th, 15th and 20th iterations. Thus, the results substantiate that PSOPA algorithm can converge successfully. Meanwhile, the figure also shows that larger swarm number in PSO will bring a slight revenue gain to the MBS. The reason is that a larger population means a more dense search at one iteration, and would assist to search a solution that get much closer to the real optimal one. But, after the population gets large enough, i.e., 12, the gain will be negligible.

4.2 Performance of PSOPA-based two-tier femtocell networks

4.2.1 Average FU's EC versus delay-QoS exponent

The relationship between the average FU's EC and the delay-QoS exponent is studied in this part. Here, we set the number of femtocells to 10. The interference constraint is -85 dBm, without loss of generality. We conduct the simulation at different delay-QoS exponent θ and the results are shown in Fig. 5. As obviously shown in Fig. 5, when θ is

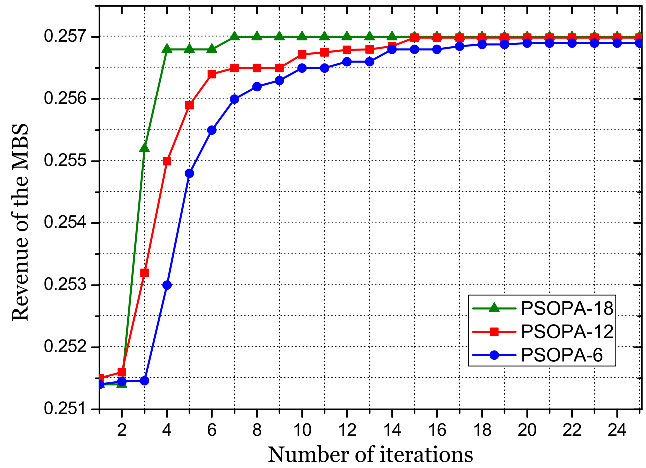

Fig. 4 Convergence Property of the PSOPA Algorithm

smaller than 10^{-6}, which implies quite a loose delay-QoS constraint, the average EC of all FUs is just the average Shannon capacity of them. As the θ gets larger, which means a more stringent delay-QoS constraint, the average EC decreases. The results coordinate with the conclusion in Section 2.

To make a comparison, we also consider the FNRAG algorithm and the EIPA algorithm. It's obviously presented in Fig. 5 that our proposed PSOPA algorithm can bring higher average EC compared with FNRAG algorithm, which is also price-based. The reason is that the interference price λ in FNRAG is determined by a try-and-error method and kept invariable, but λ in PSOPA is adjusted dynamically according to the varying of physical channel. Meanwhile, the EIPA algorithm, which divides equally the interference threshold, I_{max} dBm, to each FUs, leads to the least average EC. The reason is that it just allocates power simply and does not consider about the channel characteristics.

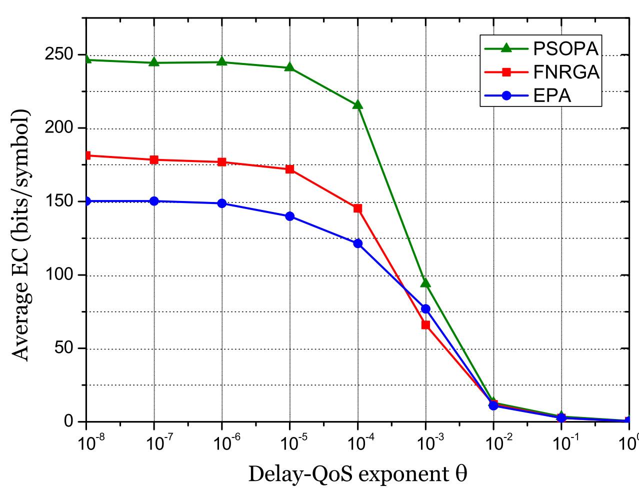

Fig. 5 Average EC vs. the Delay-QoS exponent θ

4.2.2 Average FU's EC versus interference constraint

In this part, we investigate the relationship between average EC of each FU and the interference constraint at the MBS side. Here, the number of femtocells is also set to 10, and the population of the swarm is 18. As shown in Fig. 6, the average EC of all FUs increases as the I_{max} gets larger. It is because as more interference can be bought from the MBS, the FUs can obtain more interference permission and transmit at higher power level. However, as it reach some level, i.e., -80dBm, the average EC increases little and finally stays stable, because although tolerable interference constraint at MBS is large enough, the power each FU can transmit is not infinite. In addition, with the same interference constraint, the average EC with θ being 10^{-3} is always higher than that with θ being $10^{-2.5}$, which is in accordance with the conclusion given above.

4.2.3 Influence of the number of the femtocells

Here, we investigate the influence of the number of the femtocells on the aggregate EC of all the FUs. Figure 7 shows that, under a certain interference constraint, as the number of femtocells increases, more FUs will share the scheduled subchannel, thus the aggregate EC will definitely increases. However, as the number gets larger than a certain level, the average aggregate EC stay stationary. It's because that the number of the FUs actually permitted to translate data at this subchannel is limited as a result of the restriction of the total interference at the MBS.

4.2.4 Simulation in densely deployed scenario

Finally, the relationship between the aggregate EC and the interference constraint at the MBS in the densely deployed scenario has been studied. In this scenario, we consider that

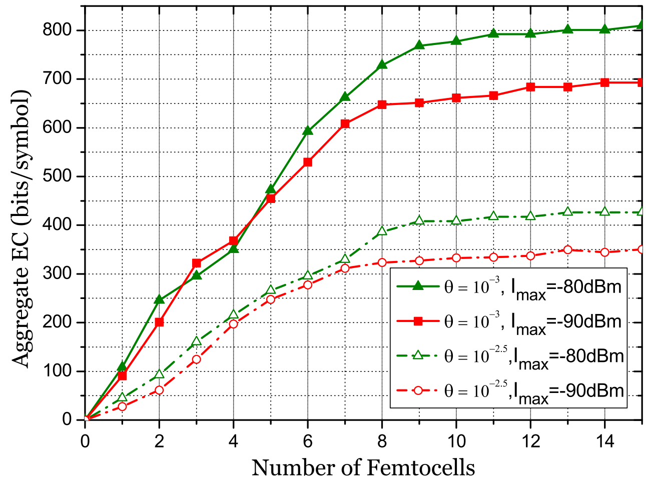

Fig. 7 Aggregate EC vs. Number of Femtocells

the total inter-femtocell interference at $B_k (k \in \mathcal{K})$ is seriously restricted within δ, which is confirmed by the degree of density in the practical circumstance. The number of the femtocells is 60, and all the delay-QoS exponents are set to 10^{-3}. Simulation results, shown in Fig. 8, demonstrate that, similar to the sparsely deployed scenario, the aggregate EC of all FUs grows larger with the increase of the interference constraint at the MBS, and it will stay stable when the interference constraint arrives at a certain degree, such as -80 dBm. Meanwhile, in ideal case, with $\delta = 0$, the aggregate EC achieves the highest but it will decrease with the increase of δ.

Additionally, the aggregate EC at different number of femtocells in femtocell networks with inter-femtocell interference restricted has also been studied. As shown in Fig. 9, when the number of femtocell increases, the aggregate EC will increase at first, but stay steady when the number gets

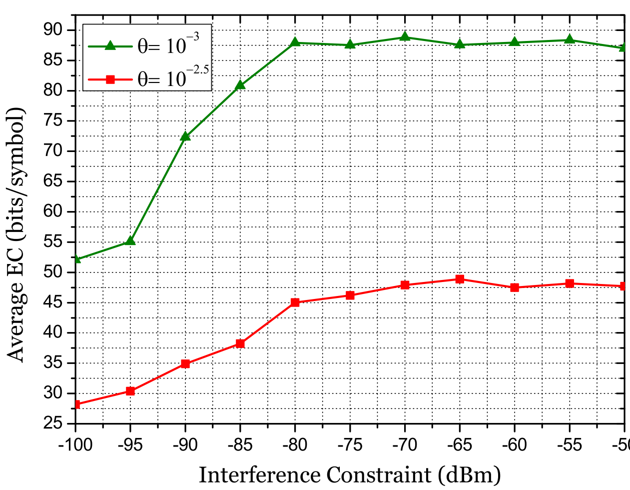

Fig. 6 Average EC vs. Interference Constraint

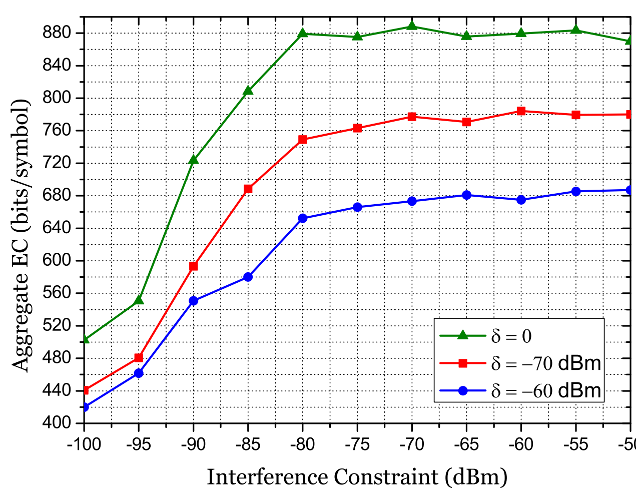

Fig. 8 Aggregate EC vs. Interference Constraint in densely deployed Femtocell Networks

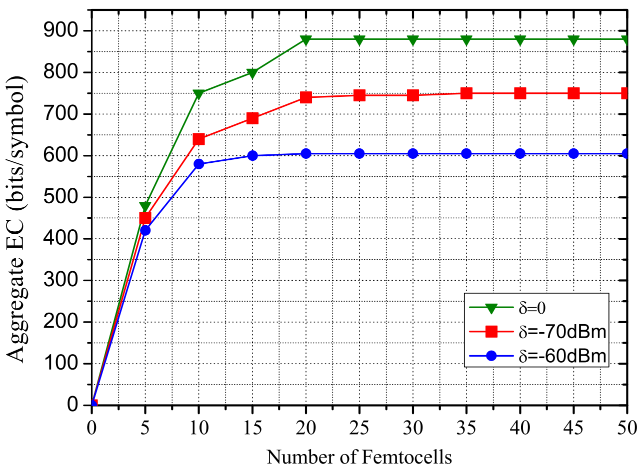

Fig. 9 Aggregate EC vs. Number of Femtocells with different δ

large enough, because of the interference constraint at the MBS.

5 Conclusions

As a result of the random and co-channel deployment of femtocells, the macrocell will suffer serious cross-tier interference from femtocells in two-tier femtocell networks. In addition, with the explosive popularity of smart terminals, the wireless networks have loaded a mount of data services with diverse delay QoS requirements. In order to mitigate the cross-tier interference at MBS, we have adopted a price-based power control strategy, in which the MBS protects itself by pricing the interference from FUs. Additionally, to guarantee the statistical delay QoS for each FU, the effective capacity has been introduced into the utility function of each FU. Based on the mathematically analysis of a Stackelberg game aimed at joint utility maximization of the MBS and the FUs, we have proposed an efficient power allocation algorithm, named PSOPA. Finally, simulation results have shown that our proposed PSOPA algorithm can not only improve significantly the average effective capacity of each FU and guarantee their statistical delay QoS, but also converge successfully.

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References

1. (2014). Cisco, White Paper
2. Chandrasekhar V, Andrews J, Gatherer A (2008) IEEE Commun Mag 46(9):59. doi:10.1109/MCOM.2008.4623708
3. Andrews J, Claussen H, Dohler M, Rangan S, Reed M (2012) IEEE J Sel Areas Commun 30(3):497. doi:10.1109/JSAC.2012.120401
4. Bennis M, Perlaza S (2011). In: 2011 IEEE International Conference on Communications (ICC), pp 1–5. doi:10.1109/icc.2011.5962649
5. Lien SY, Tseng CC, Chen KC, Su CW (2010). In: 2010 IEEE International Conference on Communications (ICC), pp 1–6. doi:10.1109/ICC.2010.5502784
6. Sun Y, Jover R, Wang X (2012) IEEE Trans Wirel Commun 11(2):614. doi:10.1109/TWC.2011.120511.101794
7. Jo HS, Mun C, Moon J, Yook JG (2009) IEEE Trans Wirel Commun 8(10):4906. doi:10.1109/TWC.2009.080457
8. Saraydar C, Mandayam NB, Goodman D (2001) IEEE J Sel Areas Commun 19(10):1883. doi:10.1109/49.957304
9. Hou Y, Laurenson D (2010). In: 2010 IEEE 72nd Vehicular Technology Conference Fall (VTC 2010-Fall), pp 1–5. doi:10.1109/VETECF.2010.5594154
10. Erturk M, Aki H, Guvenc I, Arslan H (2010). In: 2010 IEEE Global Telecommunications Conference (GLOBECOM 2010), pp 1–6. doi:10.1109/GLOCOM.2010.5683397
11. Ellouze R, Gueroui M, Alimi A (2011). In: 2011 IEEE Wireless Communications and Networking Conference (WCNC), pp 84–89. doi:10.1109/WCNC.2011.5779111
12. Taleb T, Ksentini A (2012). In: 2012 IEEE International Conference on Communications (ICC), pp 5146–5150. doi:10.1109/ICC.2012.6364289
13. Lin S, Tian H (2013). In: 2013 IEEE Wireless Communications and Networking Conference (WCNC), pp 649–654. doi:10.1109/WCNC.2013.6554640
14. Tsiropoulou E, Katsinis G, Vamvakas P, Papavassiliou S (2013). In: 2013 IEEE 18th International Workshop on Computer Aided Modeling and Design of Communication Links and Networks (CAMAD), pp 104–108. doi:10.1109/CAMAD.2013.6708098
15. Wu D, Negi R (2003) IEEE Trans Wirel Commun 2(4):630. doi:10.1109/TWC.2003.814353
16. Tang J, Zhang X (2007) IEEE Trans Wirel Commun 6(12):4349. doi:10.1109/TWC.2007.06031
17. Qiao D, Gursoy M, Velipasalar S (2011) IEEE Trans Commun 59(7):2006. doi:10.1109/TCOMM.2011.051311.090315
18. Xiong C, Li G, Liu Y, Chen Y, Xu S (2013) IEEE Trans Wirel Commun 12(6):3085. doi:10.1109/TWC.2013.050313.121695
19. Al Daoud A, Alpcan T, Agarwal S, Alanyali M (2008). In: 2008. CDC 2008. 47th IEEE Conference on Decision and Control, pp 1422–1427. doi:10.1109/CDC.2008.4738975
20. Economides AA, Silvester JA (1990). In: Proceedings 28th Annual Allerton Conference on Communications, Control and Computing, p 1990
21. Boyd S, Vandenberghe L (2004). In: Cambridge University Press
22. Saraydar C, Mandayam NB, Goodman D (2002) IEEE Trans Commun 50(2):291. doi:10.1109/26.983324
23. Kennedy J, Eberhart R (1995). In: IEEE International Conference on Neural Networks, 1995. Proceedings, vol 4, pp 1942–1948. doi:10.1109/ICNN.1995.488968
24. Shi Y, Eberhart R (1999). In: 1999. CEC 99. Proceedings of the 1999 Congress on Evolutionary Computation, vol 3, p 1950. doi:10.1109/CEC.1999.785511
25. Xie T, Zhang G, Xie J, Liu Y (2013). In: 2013 2nd International Symposium on Instrumentation and Measurement, Sensor Network and Automation (IMSNA), pp 489–493. doi:10.1109/IMSNA.2013.6743322
26. RAN4 G. In: R4-092042